Square root law for price impact: Empirical evidence and theory

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Summary

I. Introduction to price impact and LLOB

By definition, trading affects the shape of supply and demand

BUT HOW ?

To determine the properties of supply and demand, we need to probe it...

Numerical results [animation]

$$y_t$$

$$\rho_B \qquad\qquad\qquad\qquad\qquad \rho_A$$

Evolution in presence of a metaorder m_t, in the LLOB framework.

$$y_t = \frac{1}{\mathcal{L}} \int_0^t \frac{\mathsf{d}s\, m_s}{\sqrt{4\pi D(t-s)}} e^{-\frac{(y_t - y_s)^2}{4D(t-s)}} \tag{1}$$
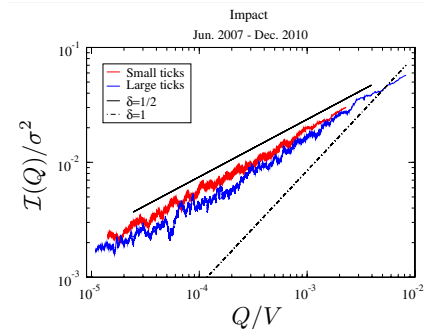
Relevance of price impact

Why is this issue relevant?

▶ **Theory (I):** Relevant, because price impact is a way to probe the supply and demand curves, so as to determine their properties;

▶ **Theory (II):** Because price impact is the mechanism through which prices absorb information encoded in trades; because it is the core ingredient of many agent-based models that aim to study price formation;

▶ **Practice (I):** Price impact is a cost for traders, which they need to accurately control in order to optimize their execution;

▶ **Practice (II):** For regulators, price impact controls stability.

Some historical results on financial markets

Evidence that dates back to 1997 (!) shows that **impact has a concave shape** (roughly) independent of:

▶ Venue

▶ Maturity

▶ Historical period

▶ Geographical area

[see Torre (1997), Almgren et al.(2003), Moro et al. (2009), Tóth et al.(2011), Gomes, Waelbroeck (2014),Bershova, Rakhlin (2013), Mastromatteo et al.(2014), X. Brokmann et al. (2014),Zarinelli et al. (2015)]

Impact
Jun. 2007 - Dec. 2010

Legend:
— Small ticks
— Large ticks
— $\delta=1/2$
-·- $\delta=1$

Axes: $\mathcal{I}(Q)/\sigma^2$ versus Q/V

[from Tóth et al (2011), impact of $\approx$ 500 000 sequences]

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$$\mathcal{I}(Q) = Y\sigma \left(\frac{Q}{V} \right)^{1/2} \quad (2)$$

$\mathcal{I}(Q)$ price change

Q executed volume

σ daily volatility

V daily traded volume

Y Y-ratio
(adimensional ~ 1)

II. General insights on Bitcoin, data and metaorders

General aspects of the Bitcoin/USD market

▶ Crypto-currency, exchanged against usual currencies on limit order books,
▶ Power law distribution of volumes traded and traders wealth,
▶ Power law distribution of returns,
▶ Unpredictable price changes.

Almost like a usual market, except...

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Almost like a usual market, except...

- ▶ One exchange (MtGox) with market share $> 80\%$ (at that time) and few correlated product/derivatives,
- ▶ Very large spread and fees,
- ▶ Few professionals (no significant HFT, market making, almost no brokerage...),
- ▶ Trading intentions are displayed much longer in advance !

Data

▶ Snapshots of the whole order book of MtGox every 10 min since 2011

▶ MtGox full trading report (7M trades) with anonymized IDs.

Metaorders
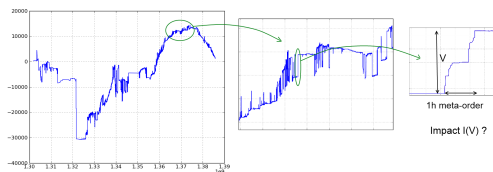
Position of a given trader vs time and zoom on a metaorder

▶ Times series decomposition rather irrelevant due to the irregular nature of the time series.

▶ Method used:
  ▶ for each trader, spot periods of inactivity (>1h)
  ▶ define the start of a metaorder as the first trade after this period
  ▶ continue until either a new inactivity period or a position reversal
  ▶ this eliminates some sequences, but also mean-reversion biases

We consider only aggressive orders to limit adverse selection biases (since we don't know the target volume to execute).

Results on metaorders

Metaorders size distribution

Metaorders duration distribution

Metaorders participation rates distribution

Average execution paths
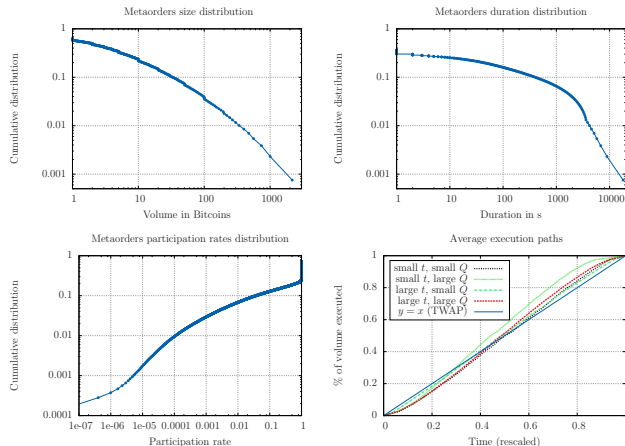
(Bottom right) Execution speed is on average constant on $[0, T] =>$ this limits selection biases and is a sign of poorly strategic behaviour regarding execution

Results on metaorders

Metaorders during a metaorder (#)

Metaorders during a metaorder (volume)
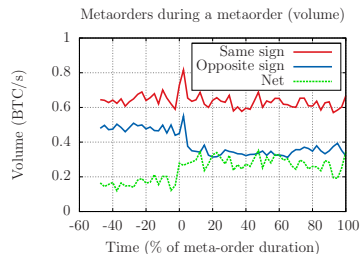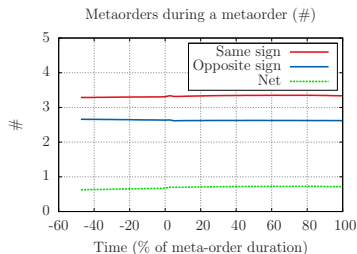
Metaorders are positively correlated with the aggressive imbalance of other traders, but the effect is not dynamical (the correlation remains constant on $[0, T]$).

Thus the impact picture will not come from some dynamical synchronization between agents (who for instance would all try to exploit the same signal at the same time, resulting in a sharper increase in price when the signal is released).

III. Market impact on the Bitcoin [Donier and Bonart, 2014]

Typical impact shape

Abrupt rise of the price to $\mathcal{I}(Q)$ then decay
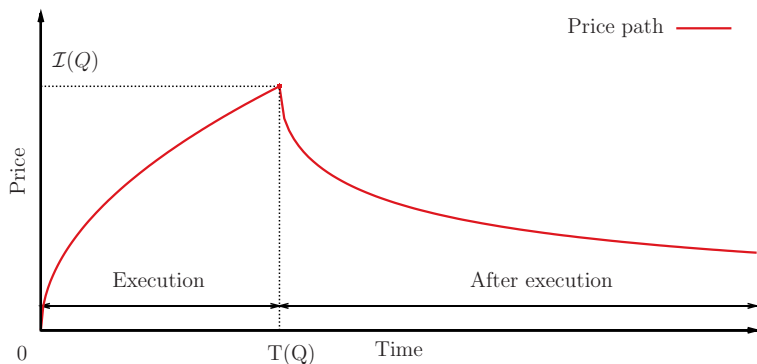
Market impact on the Bitcoin

What would we expect on the Bitcoin market?

Reminder: fees are high, market is immature, agents are amateur, EMH is not the rule on such scales...

Market impact on the Bitcoin

*Over 4 decades, impact $\mathcal{I}(Q) = \langle (p_T - p_0) \mid Q \rangle$ is square root*
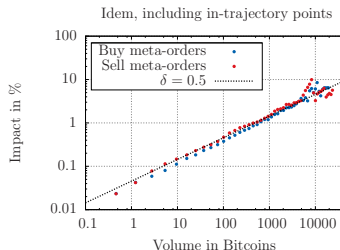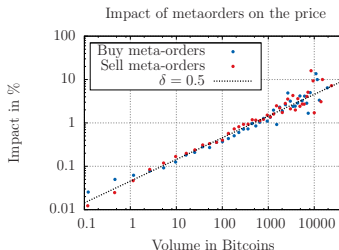
Remark: For the right plot, metaorders are regularly sub-sampled in quantiles of volumes (every 2.5%) so that every trajectory has equal weight.

Market impact on the Bitcoin

Question: is it possible that this comes from a conditioning between the executed volume and the price signal?

Market impact on the Bitcoin

▶ The whole price trajectories during impact $p_t, t = 0...T$ (for given Q, T) are square root of time (not only the end points);

▶ Thus, the square root form for the scatter plot $(\mathcal{I}(Q), Q)$ does not come from a conditioning of Q on the price signal: it is a trajectory effect;

▶ This suggests the existence of a **microscopic mechanism** to produce this shape.

Trading rate: 10 BTC/s
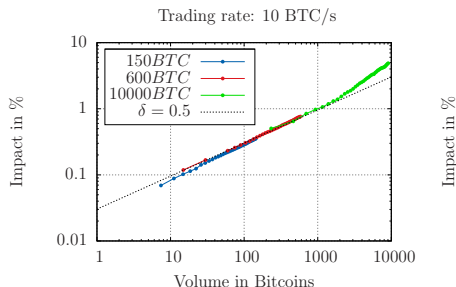
Bid, ask, traded price: What relevance?
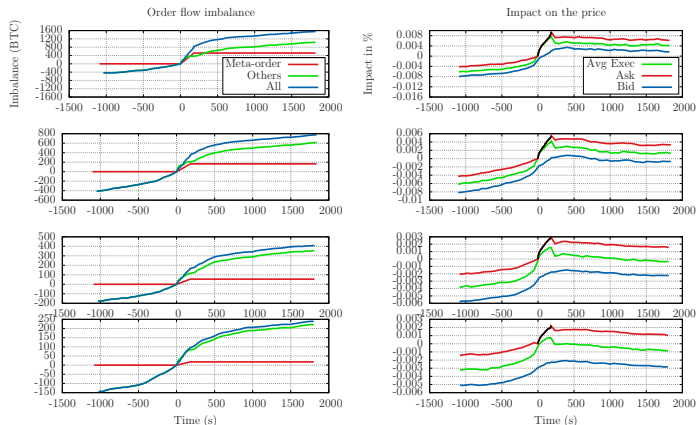
Some impact trajectories for given Q,T. The "price" means nothing here: Only the *best opposite* is relevant for our purposes and give a square root, which supports mechanical theories of impact as LLOB.
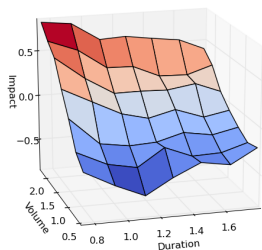
Opposite side dynamics after the trade

- ▶ After the execution is completed, the opposite side reverts;
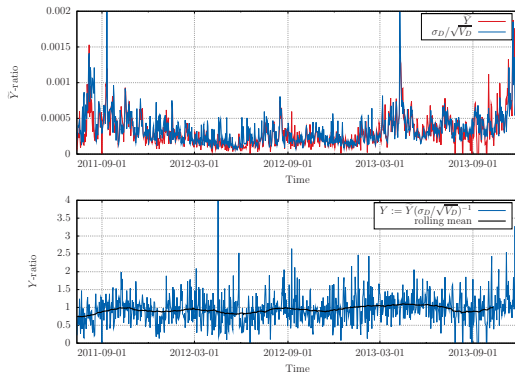- ▶ For isolated trades, it almost reverts to the initial price *[see also Brokmann et al., 2014]*.
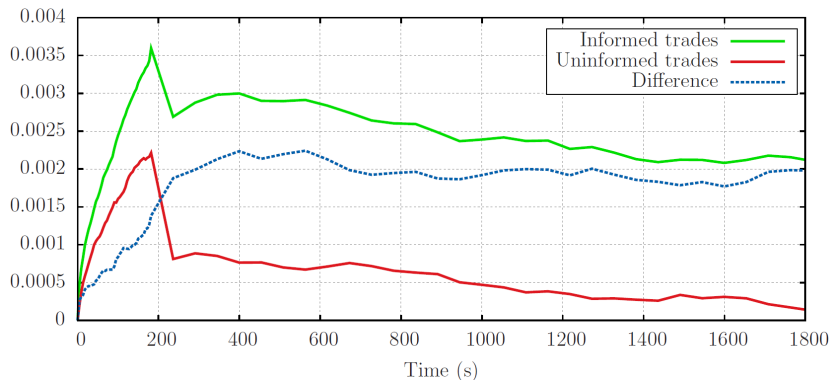
Execution speed

Cost increases with execution speed for isolated metaorders

Remark: For non-isolated metaorders, changing the execution horizon T changes the amount of correlation with the markets: we would observe the wrong effect!

Impact pre-factor

(Top) Impact pre-factor $\mathcal{I}(Q)/\sqrt{Q}$ vs usual normalization $\sigma_D/\sqrt{V_D}$ (Bottom) Residual Y-ratio

Lessons from the Bitcoin study

What did we learn here?

- ▶ A constant pressure on the price lifts it as a square root of time;

- ▶ The square root holds at all scales, in particular far below the spread and the fees;

- ▶ The impact of isolated orders reverts to zero (or close); the part of the impact that appears permanent is only due to *correlation with the market overall direction*.

- ▶ Because of the microstructure of the Bitcoin market, EMH cannot be the determinant of impact. More **mechanical mechanisms** must be at stake.

- ▶ This study strongly supports the LLOB theory *[Donier, Bonart, Mastromatteo and Bouchaud, 2014]*.

IV. Zooming-out: Bubbles and crashes

We have a full description of what happens at the **microscopic scale**

CAN WE GO FURTHER ?

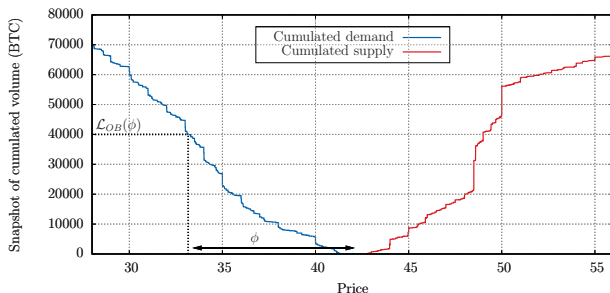Let us investigate some **macroscopic features**: Bubbles, crashes...

Liquidity on the order book fluctuates... [animation]

...which should reflect at all scales!

Macroscopic liquidity: Definition

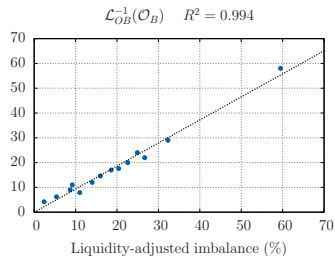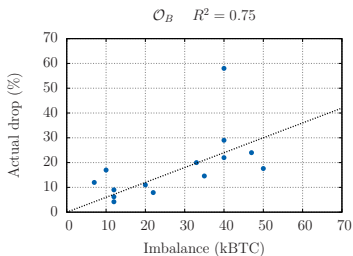Let us introduce a macroscopic definition of liquidity $\mathcal{L}_{\text{OB}}$:

$$\int_{p_t(1-\phi)}^{p_t} \mathrm{d}p\rho(p,t) := \mathcal{L}_{\text{OB}}(\phi) \,, \tag{3}$$

This definition is meaningful on the Bitcoin where liquidity is displayed long in advance (as opposed to financial markets).
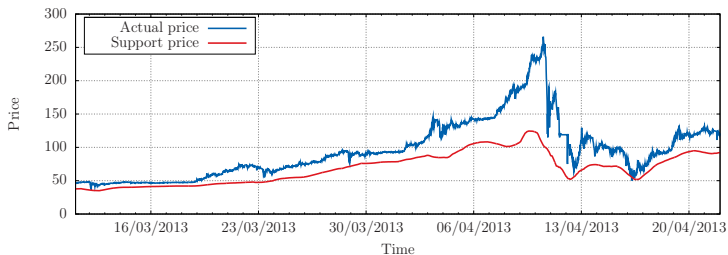
Macroscopic liquidity: Fact I

$\mathcal{L}_{OB}$ correctly predicts the amplitude of crashes:
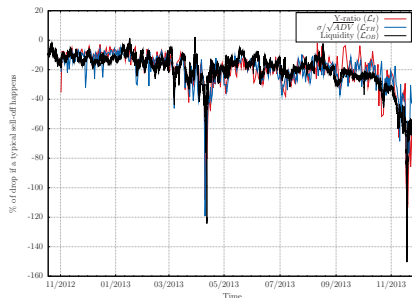
Macroscopic liquidity: Fact II

A support price can be defined as ϕ^* such that $\phi^* = \mathcal{L}_{\text{OB}}^{-1}(Q^*)$, where $Q^* := 40kBTC$ (typical large sell-off):
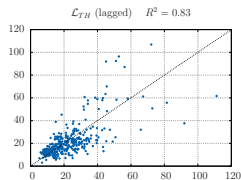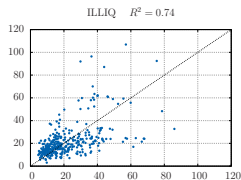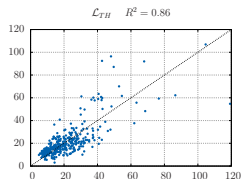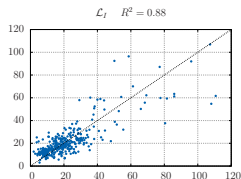
Macroscopic liquidity: Fact III

$\mathcal{L}_{\text{OB}}$ is well tracked by the theoretical and empirical impact pre-factors:

- $\mathcal{L}_{\text{I}} := \mathcal{I}(Q)/\sqrt{Q}$
- $\mathcal{L}_{\text{TH}} := \sigma_D/\sqrt{V_D}$

Macroscopic liquidity: Fact IV

The theoretical impact pre-factor $\mathcal{L}_{\mathsf{TH}} := \sigma_D/\sqrt{V_D}$ is a **good predictor of tomorrow's liquidity** (much better that Amihud's ILLIQ measure):

Conclusions: Bubbles and crashes

Based on a microscopic understanding of liquidity and price impact, we proposed a measure of liquidity $\sigma_D/\sqrt{V_D}$ (σ_D: daily volatility, V_D: daily volume) that

▶ Is publicly available,

▶ Detects bubbles,

▶ Correctly predicts the amplitude of potential crashes,

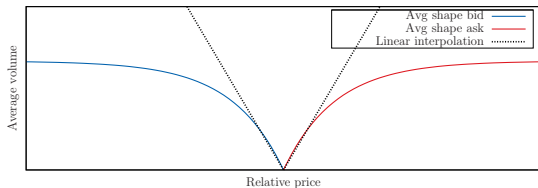▶ Largely outperforms Amihud's popular ILLIQ measure σ_D/V_D.

Conclusions: Overall

▶ Continuous trading has a universal effect on th eorder book shape.

▶ It makes it grow linearly next to the price.

▶ A microscopic understanding of liquidity allows for a prediction of extreme macroscopic events (bubbles, crashes...).
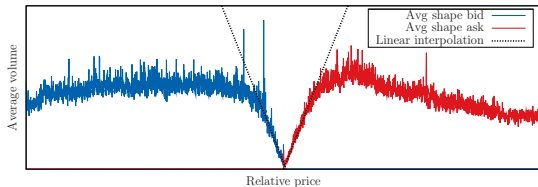
One last picture...

Order book...

In the LLOB
theory

On the Bitcoin

References

References

▶ J Donier, J Bonart, *A Million Metaorder Analysis of Market Impact on the Bitcoin*, arXiv 1412.4503 (2014)

▶ X Brokmann, E Serie, J Kockelkoren, J-P Bouchaud, *Slow decay of impact in equity markets*, arXiv 1407.3390 (2014)

▶ B.Tòth, Y Lempérière, C Deremble, J de Lataillade, J Kockelkoren, J-P Bouchaud, *Anomalous price impact and th critical nature of liquidity in financial markets*, Phys. Rev. X (2011)

▶ I Mastromatteo, B.Tòth, J-P Bouchaud, *Agent-based models for latent liquidity and concave price impact*, Phys. Rev. E **89** (2014)

▶ I Mastromatteo, B.Tòth, J-P Bouchaud, *Anomalous impact in reaction-diffusion models*, Phys. Rev. Lett. (2014)

▶ J Donier, J Bonart, I Mastromatteo, J-P Bouchaud, *A fully consistent, minimal model for non-linear market impact*, arXiv 1412.0141 (2014)